The Transient Response of a Dipole Antenna Using a Summation Approach in Technique (SAT)

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***Abstract*—A novel summation approach in technique (SAT) improving on the usual computation by multiple reflections to find the transient response was presented in a previous paper. This technique leads to accurate answers consisting of relatively few terms. The transient response at large values of time can be found without need to know explicitly and laboriously add all multiple reflections for the whole preceding range of time involved in producing the response. In this paper, the transient response for a realistically modeled pulse generator exciting a dipole antenna modeled as a distributed parameter load is investigated using a generalization of the SAT. A detailed comparison between this technique and the standard superposition of multiple reflections is presented.**

***Index Terms*—Computational electromagnetics, EMC, lightning, pulsed radiation, time-domain techniques, transients.**

I. INTRODUCTION

It is well known that time- domain techniques are very useful to compute the response of structures excited by transient electromagnetic signals. The transient response to generated pulses for different kinds of loads has been studied extensively in the literature [2]–[12]. The transient response may be formulated, in general, as an infinite number of successively delayed waves. It may be computationally difficult to accurately predict the transient response at larger values of time by the conventional method of adding all multiple reflections involved in producing the response at the whole previous range of time.

In [1], a new summation approach in technique (SAT) to find the transient response which does not require evaluation of all of these multiple reflections and leads to exact answers has been presented. In this paper, this technique is applied to study the transient response of a dipole antenna positioned as a load on an initially charged lossless transmission line operating as pulse generator.

In Section II, a review of SAT is presented. In Section III, The transient input current and voltage responses for a pulse generator loaded with a dipole antenna using SAT are investigated. Overall conclusions are presented in Section IV.

II. A REVIEW OF SAT

The summation approach in technique (SAT) to find the transient response [1] is based on the following readily demonstrated character of the Laplace transform solution for a given circuit configuration. Typically, this transform of $v(t)$, $V(s)$ is expanded as a sum of terms with an apparent singularity at s = 0. Since the limit as $t \rightarrow \infty$ of $v(t)$ = 0, the Laplace transform is, in fact, regular at s = 0. The terms of the expansion may contain other apparent singular points of individual terms $s_1, s_2, \cdots$, when the transform is, in fact, an analytic function at these points. The *net* residue of the inversion integral at such apparent singular points of the individual terms of $V(s)$ must vanish. The same may be said for $i(t)$ and its transform $I(s)$.

This character of $V(s)$ and $I(s)$ may be used to find expressions for the exact transient $v(t)$ and $i(t)$ responses consisting of relatively few terms (no matter what the value of time) compared to number of terms required in the usual computation by straightforward addition of multiple reflections. In Fig. 1, a base band-band pulse may be generated by discharging an initially charged lossless transmission line through a load. The pulse is initiated by closing a switch. In this system, the line is terminated in a load Z_L. Consider an initially charged section of lossless transmission line, $v(z,0) = V_0$, $0 < z < d$. One way to get the transient response for this system is to solve the wave equation. The Laplace transform of the wave equation with respect to the time variable

$$\frac{d^2V(z,s)}{dz^2} = L_0C_0[s^2V(z,s) - sv(z,0) - v'(z,0)]. \quad (1)$$

Given the initial values $v(z,0) = V_0$, $v'(z,0)$ = 0 and the boundary conditions $I(d,s)$ = 0, $V(0,s) = -Z_L(s)I(0,s)$, the normalized transient voltage response at the load may be expressed in the form

$$V(0,s)/V_0 = \frac{1}{s} - \frac{1}{s[Z_L(s)+1]}\,\frac{1}{[1-\rho(s)\exp(-2s)]} - \frac{1}{s[Z_L(s)+1]}\exp(-2s)\frac{1}{[1-\rho(s)\exp(-2s)]} \quad (2)$$

where $\rho(s) = [Z_L(s)-1]/[Z_L(s)+1]$ is the reflection coefficient, $Z_L(s)$ is normalized to the characteristic impedance of the transmission line $Z_0 = \sqrt{L_0/C_0}$, and the time variable is normalized so that the time for a single transit of the section of transmission line d $\sqrt{L_0C_0} = 1$.

Assume that $V(0,s)/V_0$ is an analytic function at s = 0, and $s_1, s_2, \cdots$, the zeros of $[Z_L(s)+1]$. Therefore the residues at each of these points in the s plane of

$$\text{Res}[V(0,s)/V_0]\exp(st)|_{s=0,s1,s2,\cdots} = 0. \quad (3)$$

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The reader is referred to [1] for the steps leading to the normalized transient voltage response for a general lumped complex load in the system shown in Fig. 1, in the form

$$\begin{aligned} v(0,t)/V_0 &= \sum \mathrm{Res} \\ &\quad \cdot \left\{ \frac{[\rho(s)\exp(-2s)]^{n-2}\exp(st)}{s[Z_L(s)+1][1-\rho(s)\exp(-2s)]} \right\}_{s=0,s1,s2,\cdots} \\ &\quad + \sum \mathrm{Res} \\ &\quad \cdot \left\{ \frac{[\rho(s)\exp(-2s)]^{n-2}\exp(-2s)\exp(st)}{s[Z_L(s)+1][1-\rho(s)\exp(-2s)]} \right\}_{s=0,s1,s2,\cdots} \\ &\quad - L^{-1}\left[\frac{\{\rho(s)\exp(-2s)\}^{n-2}}{s[Z_L(s)+1]} \right] \\ &\quad - L^{-1}\left[\frac{\exp(-2s)\{\rho(s)\exp(-2s)\}^{n-2}}{s[Z_L(s)+1]} \right] \\ &\quad - L^{-1}\left[\frac{\{\rho(s)\exp(-2s)\}^{n-1}}{s[Z_L(s)+1]} \right] \end{aligned} \tag{4}$$

where in the summations of residue terms $n = n(t) = [(t+1)/2 + 1/2]$, $n \geq 2$, (n is the largest integer number less than n contained in the number within the brackets) and $2(n-1) < t < 2n$. In (4), L^{-1}, means the inverse Laplace transform.

Equation (4) is applicable for a general lumped circuit parameter load, i.e., $Z_L(s)$ and $\rho(s)$, is not function of $\exp(\pm s)$. Consequently, the transient response obtained by using (4) has jump points every two units of normalized values of time. If $Z_L(s) = f[\exp(\pm s)]$, this may lead to jump points depending on the nature of this function. As will be shown in the next section, the SAT to find the transient response for this kind of load can nevertheless be implemented by appropriately carrying out the general steps which led from (2)–(4) in the instance of a lumped parameter load.

III. TRANSIENT RESPONSE OF A DIPOLE ANTENNA USING SAT

In this section, SAT is applied to find the transient current and voltage responses of a dipole antenna modeled as a distributed parameter load excited by the pulse generator shown in Fig. 1, where the load Z_L is equivalent to the input impedance of the antenna. The results obtained using this technique are compared with those obtained by superposition of multiple reflections.

For the system shown in Fig. 1, the normalized transient current response at the load may be expressed in the form

$$I(0,s)/I_0 = -\frac{1}{s[Z_L(s)+1]}[1-\exp(-2s)] \cdot \frac{1}{[1-\rho(s)\exp(-2s)]} \tag{5}$$

where $I_0 = V_0/Z_0$.

The transmission line representation of an antenna has been extensively studied in [3], [13]–[15]. The authors treat the antenna as an open-circuit transmission line that is open-circuited at the end. Fig. 2 illustrates the representation of a center-fed dipole antenna as an open-circuit transmission line. The equivalent transmission line has a length equal to the half-length H of the dipole antenna. The diameter of the antenna or transmission line conductor is $2a$. An approximation of the input impedance of this antenna, [14], is

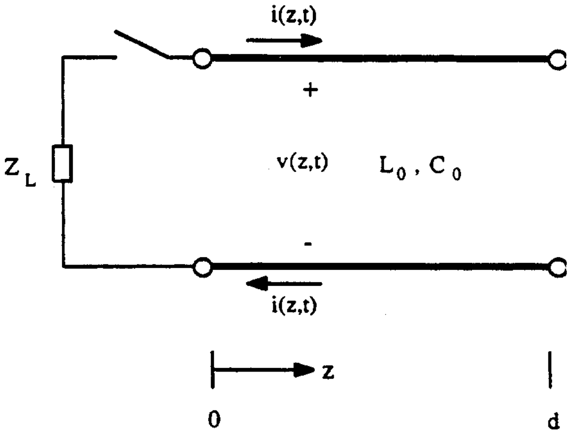

Fig. 1. Initially charged lossless transmission line.

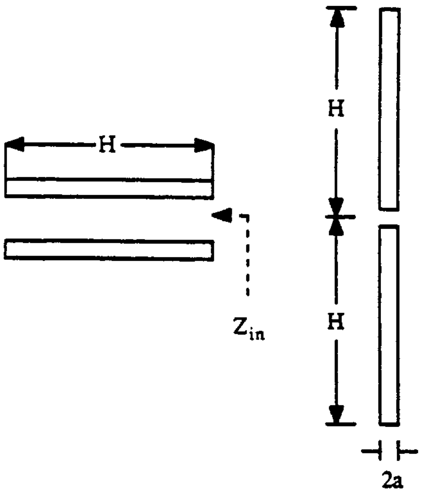

Fig. 2. The antenna as an open-circuit transmission line.

$$Z_{\mathrm{in}} = Z_c(av)\coth \gamma H = Z_L \tag{6}$$

where $\gamma = \alpha + j\beta$ is the complex propagation constant for the line. Its imaginary part $\beta = 2\pi/\lambda$ is the phase-shift constant, and its real part α is an attenuation constant introduced to account for radiation and λ is the wave length.

Consider the particular configuration in which the average characteristic impedance, $Z_c(av)$, of the dipole antenna matched to the characteristic impedance of the pulse generator Z_0 and $H = d/2$. Then

$$Z_L(s) = \frac{\exp(\psi)\exp(s)+1}{\exp(\psi)\exp(s)-1} \tag{7}$$

where $\psi = \alpha d$ and s is the Laplace transform variable. Consequently, the reflection coefficient will be

$$\rho(s) = \exp(-\psi)\exp(-s). \tag{8}$$

A. *The Transient Current Response of this Dipole Antenna*

Substituting (7) and (8) into (5), one may get

$$\begin{aligned} I(0,s)/I_0 = -\frac{1}{2}\,\frac{1}{s}\,\frac{1}{[1-\exp(-\psi)\exp(-3s)]} \\ \cdot [1-\exp(-\psi)\exp(-s)][1-\exp(-2s)]. \end{aligned} \tag{9}$$

Since (9) is an analytic function at s = 0, therefore the net residue of the inversion integral at s = 0 must vanish,

$$\mathrm{Res}[I(0,s)/I_0]\exp(st)|_{s=0} = 0 \tag{10}$$

Expanding the term $1/[1-\exp(-\psi)\exp(-3s)]$, (9) can be written in the form

$$\begin{aligned} I(0,s)/I_0 = &-\frac{1}{2}\,\frac{1}{s}\{1-\exp(-\psi)\exp(-s)+\exp(-\psi) \\ &\cdot \exp(-3s)-\exp(-2\psi)\exp(-4s)+\exp(-2\psi) \\ &\cdot \exp(-6s)-\exp(-3\psi)\exp(-7s)+\exp(-3\psi) \\ &\cdot \exp(-9s)-\exp(-4\psi)\exp(-10s)+\cdots \\ &+\exp[-(n-2)\psi]\exp[-3(n-2)s] \\ &-\exp[-(n-1)\psi]\exp[-(3n-5)s] \\ &+\exp[-(n-1)\psi]\exp[-3(n-1)s] \\ &-\exp(-n\psi)\exp[-(3n-2)s]\}-\frac{1}{2}\,\frac{1}{s} \\ &\cdot [1-\exp(-\psi)\exp(-s)]\frac{1}{[1-\exp(-\psi)\exp(-3s)]} \\ &\cdot [\exp(-\psi)\exp(-3s)]^n+\frac{1}{2}\,\frac{1}{s}\{\exp(-2s) \\ &-\exp(-\psi)\exp(-3s)+\exp(-)\exp(-5s) \\ &-\exp(-2\psi)\exp(-6s)+\exp(-2\psi)\exp(-8s) \\ &-\exp(-3\psi)\cdot\exp(-9s)+\exp(-3\psi) \\ &\cdot \exp(-11s)-\exp(-4\psi)\exp(-12s) \\ &+\cdots+\exp[-(n-2)\psi]\exp[-(3n-4)s] \\ &-\exp[-(n-1)\psi]\exp[-3(n-1)s] \\ &+\exp[-(n-1)\psi]\exp[-(3n-1)s] \\ &-\exp(-n\psi)\exp(-3ns)\}+\frac{1}{2}\,\frac{1}{s} \\ &\cdot [1-\exp(-\psi)\exp(-s)]\exp(-2s) \\ &\cdot \frac{1}{[1-\exp(-\psi)\exp(-3s)]}[\exp(-\psi)\exp(-3s)]^n \end{aligned} \tag{11}$$

where the term, $1/[1-\exp(-\psi)\exp(-3s)][\exp(-\psi)\exp(-3s)]^n$, is the remainder of the series. Let

$$\begin{aligned} F_1(s) = &-\frac{1}{2}\,\frac{1}{s}\{1-\exp(-\psi)\exp(-s)+\exp(-\psi)\exp(-3s) \\ &-\exp(-2\psi)\exp(-4s)+\exp(-2\psi)\exp(-6s) \\ &-\exp(-3\psi)\exp(-7s)+\exp(-3\psi)\exp(-9s) \\ &-\exp(-4\psi)\exp(-10s)+\cdots+\exp[-(n-2)\psi] \\ &\cdot \exp[-3(n-2)s]-\exp[-(n-1)\psi] \\ &\cdot \exp[-(3n-5)s]+\exp[-(n-1)\psi] \\ &\cdot \exp[-3(n-1)s]-\exp(-n\psi)\exp[-(3n-2)s]\} \\ &-\frac{1}{2}\,\frac{1}{s}[1-\exp(-\psi)\exp(-s)] \\ &\cdot \frac{1}{[1-\exp(-\psi)\exp(-3s)]}[\exp(-\psi)\exp(-3s)]^n \end{aligned} \tag{12}$$

$$\begin{aligned} F_2(s) = &\frac{1}{2}\,\frac{1}{s}\{\exp(-2s)-\exp(-\psi)\exp(-3s) \\ &+\exp(-\psi)\exp(-5s)-\exp(-2\psi)\exp(-6s) \\ &+\exp(-2\psi)\exp(-8s)-\exp(-3\psi)\exp(-9s) \\ &+\exp(-3\psi)\exp(-11s)-\exp(-4\psi)\exp(-12s) \\ &+\cdots+\exp[-(n-2)\psi]\exp[-(3n-4)s] \\ &-\exp[-(n-1)\psi]\exp[-3(n-1)s] \\ &+\exp[-(n-1)\psi]\exp[-(3n-1)s]-\exp(-n\psi) \\ &\cdot \exp(-3ns)\}+\frac{1}{2}\,\frac{1}{s} \\ &\cdot [1-\exp(-\psi)\exp(-s)]\exp(-2s) \\ &\cdot \frac{1}{[1-\exp(-\psi)\exp(-3s)]}[\exp(-\psi)\exp(-3s)]^n. \end{aligned} \tag{13}$$

One can find the inverse transform by evaluating the appropriate inversion integral contributions due to the residues at s = 0 only

$$\begin{aligned} &\mathrm{Res}F_1(s)\exp(st)|_{s=0} \\ &\quad = -\frac{1}{2}\{U(t)-\exp(-\psi)U(t-1)+\exp(-\psi)U(t-3) \\ &\qquad -\exp(-2\psi)U(t-4)+\exp(-2\psi)U(t-6) \\ &\qquad -\exp(-3\psi)U(t-7)+\exp(-3\psi)U(t-9) \\ &\qquad -\exp(-4\psi)U(t-10)+\cdot+\exp[-(n-2)\psi] \\ &\qquad \cdot U[t-3(n-2)]-\exp[-(n-1)\psi] \\ &\qquad \cdot U[t-(3n-5)]+\exp[-(n-1)\psi]U[t-3(n-1)] \\ &\qquad -\exp(-n)U[t-(3n-2)]\}-\frac{1}{2}\mathrm{Res}\frac{1}{s} \\ &\qquad \cdot [1-\exp(-\psi)\exp(-s)]\frac{1}{[1-\exp(-\psi)\exp(-3s)]} \\ &\qquad \cdot [\exp(-\psi)\exp(-3s)]^n\exp(st)|_{s=0} \end{aligned} \tag{14}$$

$$\begin{aligned} &\mathrm{Res}F_2(s)\exp(st)|_{s=0} \\ &\quad = \frac{1}{2}\{U(t-2)-\exp(-\psi)U(t-3)+\exp(-\psi)U(t-5) \\ &\qquad -\exp(-2\psi)U(t-6)+\exp(-2\psi)U(t-8) \\ &\qquad -\exp(-3\psi)U(t-9)+\exp(-3\psi)U(t-11) \\ &\qquad -\exp(-4\psi)U(t-12)+\cdots+\exp[-(n-2)\psi] \\ &\qquad \cdot U[t-(3n-4)]-\exp[-(n-1)\psi]U[t-3(n-1)] \\ &\qquad +\exp[-(n-1)\psi]U[t-(3n-1)]-\exp(-n\psi) \\ &\qquad \cdot U(t-3n)\}+\frac{1}{2}\mathrm{Res}\frac{1}{s}[1-\exp(-\psi)\exp(-s)] \\ &\qquad \cdot \exp(-2s)\frac{1}{[1-\exp(-\psi)\exp(-3s)]} \\ &\qquad \cdot [\exp(-\psi)\exp(-3s)]^n\exp(st)|_{s=0}. \end{aligned} \tag{15}$$

We must have

$$\begin{aligned} &\mathrm{Res}[I(0,s)/I_0]\exp(st)|_{s=0} \\ &\quad = \mathrm{Res}\ F_1(s)\exp(st)|_{s=0}+\mathrm{Res}\ F_2(s)\exp(st)|_{s=0} = 0. \end{aligned} \tag{16}$$

Substituting (14) and (15) into (16) and replacing n by $(n-2)$ for convenience yields the canonical result.

$$\begin{aligned}
&\frac{1}{2}\ \mathrm{Res}\ \frac{1}{s}[1-\exp(-\psi)\exp(-s)]\frac{1}{[1-\exp(-\psi)\exp(-3s)]}\\
&\quad\cdot[\exp(-\psi)\exp(-3s)]^{n-2}\exp(st)|_{s=0}\\
&\quad-\frac{1}{2}\ \mathrm{Res}\ \frac{1}{s}[1-\exp(-\psi)\exp(-s)]\\
&\quad\cdot\exp(-2s)\frac{1}{[1-\exp(-\psi)\exp(-3s)]}\\
&\quad\cdot[\exp(-\psi)\exp(-3s)]^{n-2}\exp(st)|_{s=0}\\
&=-\frac{1}{2}\{U(t)-\exp(-\psi)U(t-1)\\
&\quad+\exp(-\psi)U(t-3)-\exp(-2\psi)U(t-4)\\
&\quad+\exp(-2\psi)U(t-6)-\exp(-3\psi)U(t-7)\\
&\quad+\exp(-3\psi)U(t-9)-\exp(-4\psi)U(t-10)\\
&\quad+\cdots+\exp[-(n-4)\psi]U[t-3(n-4)]\\
&\quad-\exp[-(n-3)\psi]U[t-(3n-11)]+\exp[-(n-3)\psi]\\
&\quad\cdot U[t-3(n-3)]-\exp[-(n-2)\psi]\\
&\quad\cdot U[t-(3n-8)]\}+\frac{1}{2}\{U(t-2)-\exp(-\psi)U(t-3)\\
&\quad+\exp(-\psi)U(t-5)-\exp(-2\psi)U(t-6)\\
&\quad+\exp(-2\psi)U(t-8)-\exp(-3\psi)U(t-9)\\
&\quad+\exp(-3\psi)U(t-11)-\exp(-4\psi)U(t-12)\\
&\quad+\cdots+\exp[-(n-4)\psi]U[t-(3n-10)]\\
&\quad-\exp[-(n-3)\psi]U[t-3(n-3)]\\
&\quad+\exp[-(n-3)\psi]U[t-(3n-7)]-\exp[-(n-2)\psi]\\
&\quad\cdot U[t-3(n-2)]\}
\end{aligned}\tag{17}$$

Excluding the remainder in (11) and taking the inverse of the Laplace transform, we can get the exact transient response for $t<3n$ as a superposition of multiple reflections in the form

$$\begin{aligned}
i(0,t)/I_0=&-\tfrac{1}{2}\{U(t)-\exp(-\psi)U(t-1)\\
&+\exp(-\psi)U(t-3)-\exp(-2\psi)U(t-4)\\
&+\exp(-2\psi)U(t-6)-\exp(-3\psi)U(t-7)\\
&+\exp(-3\psi)U(t-9)-\exp(-4\psi)U(t-10)\\
&+\cdots+\exp[-(n-3)\psi]U[t-3(n-3)]\\
&-\exp[-(n-2)\psi]U[t-(3n-8)]\}-\tfrac{1}{2}\\
&\cdot\exp[-(n-2)\psi]U[t-3(n-2)]+\tfrac{1}{2}\\
&\cdot\exp[-(n-1)\psi]U[t-(3n-5)]-\tfrac{1}{2}\\
&\cdot\exp[-(n-1)\psi]U[t-3(n-1)]+\tfrac{1}{2}\\
&\cdot\exp(-n\psi)U[t-(3n-2)]+\tfrac{1}{2}\\
&\cdot\{U(t-2)-\exp(-\psi)U(t-3)\\
&+\exp(-)U(t-5)-\exp(-2\psi)U(t-6)\\
&+\exp(-2\psi)U(t-8)-\exp(-3\psi)U(t-9)\\
&+\exp(-3\psi)U(t-11)-\exp(-4\psi)U(t-12)\\
&+\cdots-\exp[-(n-3)\psi]U[t-3(n-3)]\\
&+\exp[-(n-3)\psi]U[t-(3n-7)]\\
&-\exp[-(n-2)\psi]U[t-3(n-2)]\}\\
&+\tfrac{1}{2}\exp[-(n-2)\psi]U[t-(3n-4)]\\
&-\tfrac{1}{2}\exp[-(n-1)\psi]U[t-3(n-1)]\\
&+\tfrac{1}{2}\exp[-(n-1)\psi]U[t-(3n-1)]
\end{aligned}\tag{18}$$

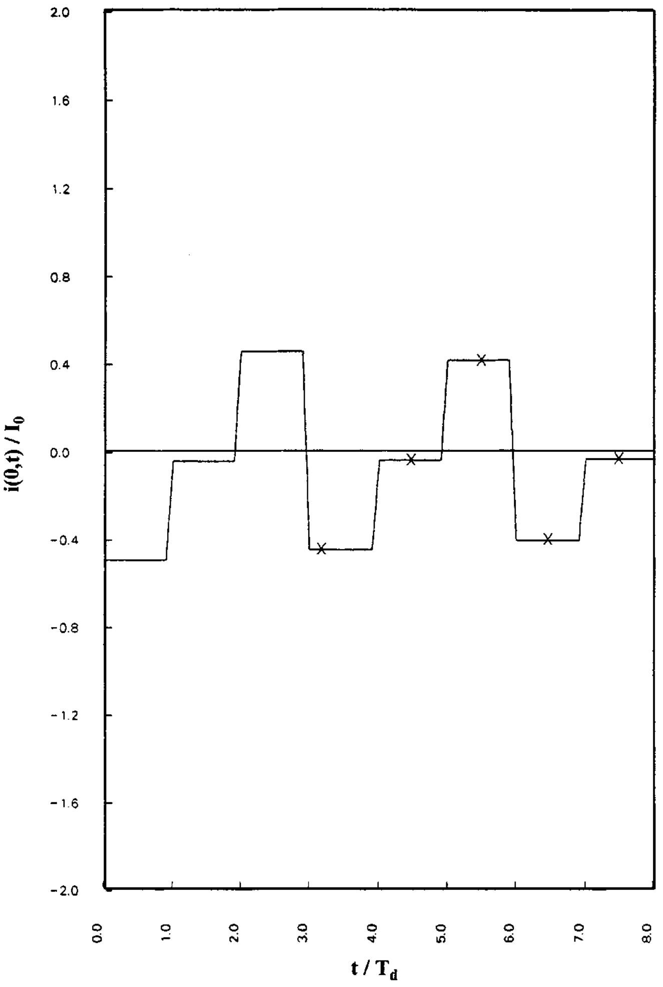

Fig. 3. The transient current response for a pulse generator loaded with a dipole antenna using the infinite succession of reflections (____) and SAT(x).

where t is normalized to $T_d=d\sqrt{L_0C_0}$, the travel time of the generator line section.

Comparing (17) and (18), we can get the transient response for $t<3n$ in the form

$$\begin{aligned}
i(0,t)/I_0=&\tfrac{1}{2}\exp[-(n-1)\psi]U[t-(3n-5)]\\
&-\exp[-(n-1)\psi]U[t-3(n-1)]\\
&+\tfrac{1}{2}\exp(-n\psi)U[t-(3n-2)]\\
&+\tfrac{1}{2}\exp[-(n-1)\psi]U[t-(3n-1)]
\end{aligned}\tag{19}$$

where $n=n(t)=[(t+1)/3+1/2]$, (n is the largest integer number less than n equal to the number in the brackets), $t<3n(t)$ and $n(t)\geq 2$.

Similarly, one can get the transient current response for $t\geq 3n$ in the form

$$i(0,t)/I_0=-\tfrac{1}{2}\exp(-n\psi)U(t-3n)\tag{20}$$

where $n=n(t)=[(t+1)/3+1/2]$, (n is the largest integer number less than n equal to the number in the brackets), $t\geq 3n(t)$ and $n(t)\geq 0$.

We can get the transient current response using the infinite succession of reflections in the time domain as

TABLE I
COMPARISON BETWEEN THE TRANSIENT RESPONSE OF A DIPOLE ANTENNA USING THE MULTIPLE REFLECTIONS CALCULATIONS AND SAT

t	multiple reflections evaluation	no. of terms	SAT evaluation				
			n(t)	t ? 3n(t)	the applied eq.	the response	no. of terms
3.2	- 0.45	2	1	t > 3n(t)	(20)	- 0.45	1
3.5	- 0.45	5	2	t < 3n(t)	(19)	- 0.45	4
4	jump point	6	2	t < 3n(t)	(19)	jump point	4
4.5	- 0.043	6	2	t < 3n(t)	(19)	- 0.043	4
5	jump point	7	2	t < 3n(t)	(19)	jump point	4
5.5	0.409	7	2	t < 3n(t)	(19)	0.409	4
6.2	- 0.409	9	2	t > 3n(t)	(20)	- 0.409	1
6.5	- 0.409	9	3	t < 3n(t)	(19)	- 0.409	4
7.5	- 0.039	10	3	t < 3n(t)	(19)	- 0.039	4
15	xxxxxxxxx	21	5	t = 3n(t)	(20)	jump point	1
15.1	xxxxxxxxx	21	5	t > 3n(t)	(20)	- 0.303	1
18.1	xxxxxxxxx	25	6	t > 3n(t)	(20)	- 0.274	1
19	xxxxxxxxx	26	7	t < 3n(t)	(19)	jump point	4

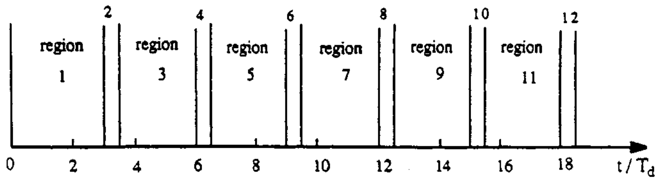

Fig. 4. A sketch clarifies the changes in parameters necessary for use of (19) and (20) to find the transient response covering certain range of time.

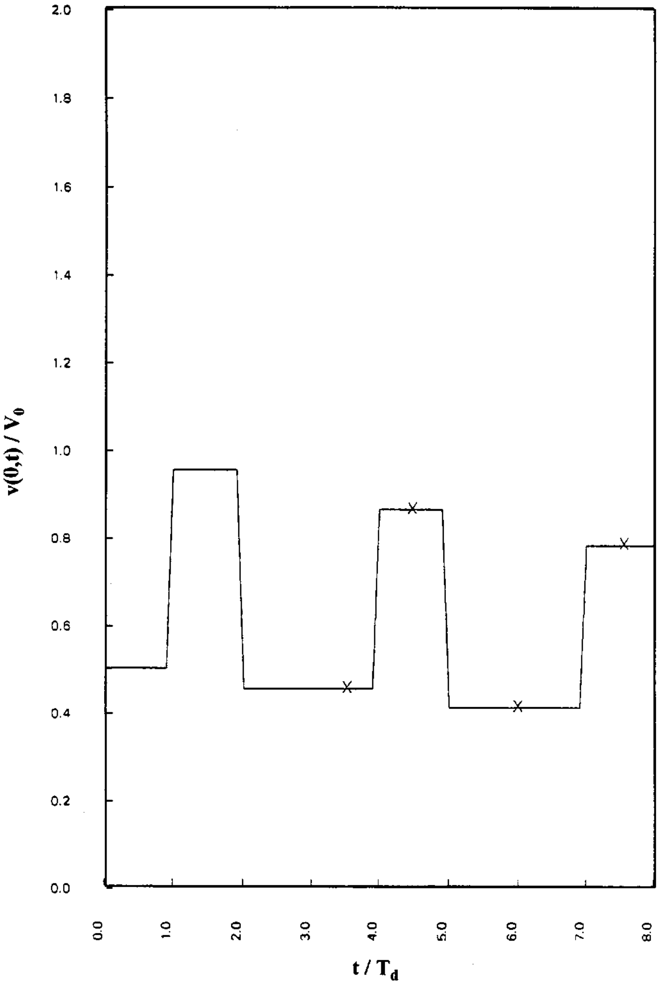

Fig. 5. The transient voltage response for a pulse generator loaded with a dipole antenna using the infinite succession of reflections (____) and SAT(x).

$$\begin{aligned} i(0,t)/I_0 = -\tfrac{1}{2}\{&U(t) - \exp(-\psi)U(t-1) - U(t-2) \\ &+ 2\exp(-\psi)U(t-3) - \exp(-2\psi)U(t-4) \\ &- \exp(-\psi)U(t-5) + 2\exp(-2\psi)U(t-6) \\ &- \exp(-3\psi)U(t-7) - \exp(-2\psi) \\ &\cdot U(t-8) + \cdots. \end{aligned} \tag{21}$$

Comparison between the results obtained by using the variation SAT represented by (19) and (20), and those obtained by using the infinite succession of reflections represented by (21) at ψ = 0.1 is shown in Fig. 3. One can see that the results are the same. Also, comparison between the required number of terms to find the transient response is shown in Table I.

One may see the difference between the number of terms required to evaluate the transient response at certain time using the infinite succession of reflections and SAT. Of course, the required number of terms will increase by increasing the time t. However, using SAT to find the transient response, the maximum number of terms required as shown in Table I is 4 terms, whatever the increased value of time. Also in Table I we show the alternative use of (19) and (20) according to the relation between t and $n(t)$. A simple sketch, which clarifies the changes in parameters necessary for use of these equations to find the transient response covering certain range of time, is shown in Fig. 4. Of course, this sketch can be extended to cover any desired range of time. However, one can notice that the use of the new SAT can not cover the range of time $0 < t < 3$ because (19) does not apply and neither does (20).

B. The Transient Voltage Response of this Dipole Antenna

For the same system shown in Fig. 1, starting with (2) and following the same steps of the variation SAT one may get an expression for the transient voltage response in the time domain in the form

$$\begin{aligned} v(0,t)/V_0 = &\tfrac{1}{2}\exp[-(n-1)\psi]U[t-(3n-5)] \\ &+ \tfrac{1}{2}\exp(-n\psi)U[t-(3n-2)] \\ &- \tfrac{1}{2}\exp[-(n-1)\psi]U[t-(3n-1)] \end{aligned} \tag{22}$$

where $n = n(t) = [(t+1)/3 + 1/2]$, ($n$ is the largest integer number less than n equal to the number in the brackets), $t < 3n(t)$ and $n(t) \geq 2$, and

$$v(0,t)/V_0 = \tfrac{1}{2}\exp(-n\psi)U(t-3n) \tag{23}$$

where $n = n(t) = [(t+1)/3 + 1/2]$, ($n$ is the largest integer number less than n equal to the number in the brackets), $t \geq 3n(t)$ and $n(t) \geq 0$.

Also, one can get the transient voltage response using the infinite succession of reflections in the form

$$\begin{aligned} v(0,t)/V_0 = U(t) - \tfrac{1}{2}\{&U(t) - \exp(-\psi)U(t-1) \\ &+ U(t-2) - \exp(-2\psi)U(t-4) \\ &+ \exp(-\psi)U(t-5) - \exp(-3\psi)U(t-7) \\ &+ \exp(-2\psi)U(t-8) + \cdots\}. \end{aligned} \qquad (24)$$

Once more, we can see in Fig. 5 that the results at $\psi = 0.1$ obtained by using the variation SAT and those obtained by using the infinite succession of reflections are the same. However, the maximum number of terms, using SAT, required to find the transient response in this case, as shown in (22), is just only three terms.

IV. CONCLUSION

The transient current and voltage responses for a pulse generator loaded with a dipole antenna modeled as a distributed constant load using the generalized variation SAT are investigated. The results obtained using the SAT compared with those obtained by using the infinite succession of reflections show that the SAT leads to an exact answer consisting of comparatively few terms. The significant value of the variation SAT becomes evident in finding the transient response at larger values of time where there is no longer the need to know all multiple reflections involved in producing the response for the whole previous range of time.

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